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Hawking Radiation of Charged Particles via Tunneling from Arbitrarily Dimensional Reissner-Nordström Black Holes

Qing-Quan Jiang,[1,2] Shu-Zheng Yang,[1] and Shuang-Qing Wu[2]

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Parikh-Wilczek's recent work, which treats the Hawking radiation as semi-classical tunneling process from the event horizon of four dimensional Schwarzshild and Reissner-Nordström black hole, indicates that self-gravitation gives a correction to the Hawking precisely thermal spectrum and the tunneling rate is related to the change of Bekenstein-Hawking, but satisfies the underlying unitary theory. In this paper, we extend the model to study the Hawking radiation of charged particles via tunneling from arbitrarily dimensional Reissner-Nordström black holes, and obtain the same result as Parikh-Wilczek's. Meanwhile, in this framework, we point out that the first law of the black hole thermodynamics is reliable and the information conservation is only suitable for the reversible process.

KEY WORDS: charged particles; energy conservation; self-gravitation; tunneling rate; information conservation.

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1. INTRODUCTION

In 1975, Hawking discovered that black holes can radiate thermally after taking the quantum effect into account, and from then on the classical 'no hair' theorem that all information about the collapsing body was lost from the outside region apart from three conserved quantities: the mass, the angular momentum, and the electric charge was discarded. Hawking regarded that the black hole radiation is created by pair of particles via tunneling from the black hole horizon as a result of the vacuum fluctuation. So there exists a tunneling process, but in this theory, the created mechanism of the tunneling barrier is unclear for us. The related references do not use the language of quantum tunneling method to

[1] Department of Physics, and College of Physical Science and Technology, Central China Normal University, Wuhan, Hubei 430079, People's Republic of China.
[2] Institute of Theoretical Physics, China West Normal University, Nanchong, Sichuan 637002, People's Republic of China.

discuss Hawking radiation, so strictly speaking, it is not the quantum tunneling method. So, to derive the factually radial spectrum from the black hole horizon, the following two difficulties must be solved: Firstly, the formed mechanism of the potential hill; Secondly, the elimination of the coordinate singularity.

For another, with the emission of thermal radiation, black holes could lose energy, shrink, and eventually evaporate away completely (Hawking, 1974). Since the radiation with a precisely thermal spectrum has no way to be recovered after black holes have disappeared completely. So it arouses a disturbing and difficult problem: what happen to information during the black hole evaporation? Hawking and Kip Torne thought that information is lost in black hole evaporation. But, John Preskill believed that information is not lost and can get out of the black hole. In 2005, Hawking changed his opinion and argued that information can indeed get out of the black hole (Hawking, 2005), which is partly based on the Parikh-Wilzcek's recent work that treats the Hawking radiation as tunneling process and applies the semi-classical method to present the actual radiation as not exactly thermal but subtle correction to the Hawking precisely thermal spectrum.

Combined with the above reasons, Parikh-Wilzcek put forward a semi-classical quantum tunneling model that implemented Hawking radiation as a tunneling process and that was initiated by Kraus and Wilczek and developed by Parikh and Wilzcek who carried out a dynamical treatment of black hole geometries. More specifically, they considered the effects of a positive energy matter shell propagating outwards through the horizon of the Schwarzshild and Reissner-Nordström black holes, and incorporated the self-gravitation correction of the radiation(Parikh and Wilczek, 2000; Parikh, 2004 and 2002). In this theory, the tunneling barrier is created by the self- gravitation action among the particles, and the location of the black hole horizon before and after the particle with energy emission can be regarded as the two turning points of the tunneling barrier. Moreover the derived result indicates that the factually radiant spectrum is not precisely thermal, but is consistent with the underlying unitary theory. Moreover this framework is so successful that it satisfies the first law of black hole thermodynamic.

However, up to now the tunneling radiation characteristics are still limited to investigate that of uncharged particles in four dimensional spacetime. But the Hawking radiation of charged particles in arbitrarily dimensional spacetime is not reported at present. The purpose of the current paper is to present a reasonable extension of the Parikh-Wilczek's semi-classical tunneling method in a spherically symmetric space-time in four dimensions to the case of arbitrarily dimensional Reissner-Nordström black holes. Moreover, Hawking radiation of charged particles via tunneling from the black hole horizon is investigated here, where a coordinate system that is well-behaved at the horizons is introduced, and the first law of the black hole thermodynamics for the reversible process is presented, and meanwhile points out that the information conservation is only suitable to the reversible process and in highly unstable evaporating black hole(irreversible process) the information loss is possible.

2. PAINLEVÉ-LIKE COORDINATE TRANSFORMATION AND THE RADIAL GEODESICS

The line element of arbitrarily dimensional Reissner-Nordström black hole is (Cai, 2002)

$$ds^2 = -f(r)\,dt_R^2 + f^{-1}(r)\,dr^2 + r^2 d\Omega_n^2, \tag{1}$$

where

$$f(r) = 1 - \frac{\omega_n M}{r^{n-1}} + \frac{n\omega_n^2 Q^2}{8(n-1)\,r^{2n-2}}, \quad \omega_n = \frac{16\pi G}{n\,\mathrm{Vol}(S^n)}, \tag{2}$$

here G is the gravitational constant in $(n+2)$ dimensions, $\mathrm{Vol}(S^n)$ denotes the volume of a unit n-sphere $d\Omega_n^2$. For general M and Q, the equation $f(r) = 0$ have two real roots, and where the larger one is the event horizon of the black hole r_+ and the smaller one is the inner horizon r_-. Throughout the following study, the units $G = c = h = 1$ are used. Obviously, at the horizons of the black hole, there exists a coordinate singularity in the metric (1), performing a Pailevé-like coordinate transformation

$$dt_R = dt \pm \frac{\sqrt{1-f(r)}}{f(r)}dr, \tag{3}$$

to remove the coordinate singularity, and we obtain Painlevé-Reissner-Nordström black hole in arbitrary dimensions as

$$ds^2 = -f(r)\,dt^2 \pm 2\sqrt{1-f(r)}dtdr + dr^2 + r^2 d\Omega_n^2, \tag{4}$$

where the plus(minus) sign denotes the space-time line element of charged outgoing (ingoing) particles across the event horizon respectively. Obviously, the Painleve-Reissner-Nordström line element in arbitrary dimensions poses many attractive properties: (i) The new form of the line element is stationary, but not static, so the time direction remains to be a Killing vector; (ii) The metric is regular at the horizons of the black hole; (iii) The time coordinate t represents the local proper time for radial free-falling observers; (iv) The measure on the surfaces of constant-time slices is the same as that of flat space-time; (v) It satisfies Landau's condition of coordinate clock synchronization. These attractive characters are very advantageous for us to investigate the tunneling radiation of the charged massive particle across the horizons and to do an explicit computation of the tunneling probability at the horizons.

Now, according to Ref. (Zhang and Zhao, 2005), let work with the new form (4) and treat charged particles as a de Broglie wave, and we obtain the radial geodesics of the charged massive particle via

$$\dot{r} = \frac{dr}{dt} = v_p = \frac{1}{2}v_g = -\frac{g_{tt}}{2g_{tr}} = \pm\frac{f(r)}{2\sqrt{1-f(r)}}, \tag{5}$$

where the plus(minus) sign denotes the radial geodesics of charged outgoing(ingoing) particles, v_p and v_g are the phase velocity and the group velocity.

3. HAWKING RADIATION OF CHARGED PARTICLES VIA TUNNELING

Now, we turn to discuss Hawking radiation of charged massive particles as a semi- classical tunneling process across the horizons. We adopt the picture of a pair of virtual particles spontaneously created just inside the horizon. The positive energy virtual particle can tunnel out and materialize as a real particle escaping classically to infinity, and its negative energy partner is absorbed by the black hole, resulting in a decrease in the mass and charge of the black hole. In our discussion, we consider the emitted particle of energy ω and charge q. If the particle's self-gravitation is taken into account, Equations (4) and (5) should be modified.

Taking energy conservation and charge conservation into consideration, and when the particle of energy ω and charge q tunnels out of the event horizon, the mass and charge parameters in Equations (4) and (5) will be replaced with $M \to M-\omega$ and $Q \to Q-q$. So the radial geodesics of the charged massive particle tunneling out from the event horizon is

$$\dot{r} = \frac{\tilde{f}(r)}{2\sqrt{1-\tilde{f}(r)}}, \tag{6}$$

where

$$\tilde{f}(r) = 1 - \frac{\omega_n(M-\omega)}{r^{n-1}} + \frac{n\omega_n^2(Q-q)^2}{8(n-1)r^{2n-2}}, \tag{7}$$

and correspondingly, the non-zero component of electromagnetic potential is

$$A_t = \frac{n}{4(n-1)}\frac{\omega_n(Q-q)}{r^{n-1}}. \tag{8}$$

When we investigate the tunneling process of a charged massive particle, the effect of the electromagnetic field outside the black hole should be taken into consideration. So the matter- gravity system consists of the black hole and the electromagnetic field outside the black hole. As $L_e = -(1/4)F_{\mu\nu}F^{\mu\nu}$ is the Lagrangian function of the electromagnetic field corresponding to the generalized coordinate described by$A_\mu = (A_t, 0, 0, 0)$, we find that the generalized coordinate A_t is an ignorable coordinate. For eliminating the freedom, the imaginary part of the action should be written as

$$\operatorname{Im} S = \operatorname{Im} \int_{t_i}^{t_f} (L - P_{A_t} \dot{A}_t) dt = \operatorname{Im} \int_{r_i}^{r_f} \left[\int_{(0,0)}^{(P_r, P_{A_t})} \left(\dot{r} dP'_r - \dot{A}_t dP'_{A_t} \right) \right] \frac{dr}{\dot{r}}, \tag{9}$$

where r_i and r_f represent the locations of the event horizon before and after the particle of energy ω and charge q tunnels out, and (P_{A_t}, P_r) are the canonical momentum conjugated to the coordinate (A_t, r) respectively. Substituting Hamilton's canonical equation of motion

$$\begin{aligned} \dot{r} &= \left. \frac{dH}{dP_r} \right|_{(r; A_t, P_{A_t})}, \quad dH|_{(r; A_t, P_{A_t})} = d(M - \omega) = -d\omega, \\ \dot{A}_t &= \left. \frac{dH}{dP_{A_t}} \right|_{(A_t; r, P_r)}, \quad dH|_{(A_t; r, P_r)} = A_t d(Q - q) = -\frac{n}{4(n-1)} \frac{\omega_n (Q - q)}{r^{n-1}} dq, \end{aligned} \tag{10}$$

into Equation (9), and switching the order of integration yield the imaginary part of the action

$$\begin{aligned} \operatorname{Im} S &= \operatorname{Im} \int_{r_i}^{r_f} \int_{(M,Q)}^{(M-\omega, Q-q)} \frac{1}{\dot{r}} \left[d\left(M - \omega'\right) - \frac{n}{4(n-1)} \frac{\omega_n \left(Q - q'\right)}{r^{n-1}} d(Q - q') \right] dr \\ &= \operatorname{Im} \int_{(M,Q)}^{(M-\omega, Q-q)} \int_{r_i}^{r_f} \frac{2\sqrt{1 - f'(r)}}{f'(r)} \left[d(M - \omega') \right. \\ &\quad \left. - \frac{n}{4(n-1)} \frac{\omega_n \left(Q - q'\right)}{r^{n-1}} d(Q - q') \right] dr \end{aligned} \tag{11}$$

where

$$f'(r) = 1 - \frac{\omega_n (M - \omega')}{r^{n-1}} + \frac{n \omega_n^2 (Q - q')^2}{8(n-1) r^{2n-2}} = \frac{1}{r^{2n-2}} (r - r'_+)(r - r'_-),$$

in which, the mass and charge parameters in r'_+ and r'_- are $M - \omega'$ and $Q - q'$ respectively. The above integral can be evaluated by deforming the contour around the single pole $r = r'_+$ at the event horizon. Doing the r integral first, we find

$$\begin{aligned} \operatorname{Im} S = -\pi \int_{(M,Q)}^{(M-\omega, Q-q)} & \frac{2 r_+^{\prime 2n-2}}{(r'_+ - r'_-)} \\ & \times \left[d(M - \omega') - \frac{n}{4(n-1)} \frac{\omega_n (Q - q')}{r_+^{\prime(n-1)}} d(Q - q') \right]. \end{aligned} \tag{12}$$

As the identity

$$\left(r'_+ - r'_-\right) \frac{1}{\omega_n r_+^{\prime(n-1)}} dr'_+ = d(M - \omega') - \frac{n}{4(n-1)} \frac{\omega_n (Q - q')}{r_+^{\prime(n-1)}} d(Q - q'), \tag{13}$$

we can easily finish the integration and obtain

$$\operatorname{Im} S = -\pi \int_{r_i}^{r_f} \frac{2}{\omega_n} r_+'^{n-1} dr_+' = -\frac{2\pi}{n\omega_n}\left(r_f^n - r_i^n\right). \tag{14}$$

Since the event horizon and the infinite red-shift surface are coincident with each other, the geometrical optical limit become an especially reliable approximation and the semi-classical WKB approximation can be used. In WKB approximation, the probability of tunneling is related to the imaginary part of the action via (Jiang and Wu, 2006; Jiang *et al.,* 2006; Yang *et al.,* 2005; Hemming and Keski-Vakkuri, 2000; Medved, 2002; Kraus and Keski-Vakkuri, 1997; Yang, 2005; Han, 2005)

$$\Gamma \sim e^{-2\operatorname{Im} S}, \tag{15}$$

So the tunneling rate at the event horizon is

$$\Gamma \sim e^{-2\operatorname{Im} S} = e^{\frac{4\pi}{n\omega_n}\left(r_f^n - r_i^n\right)} = e^{\Delta S_{BH}}, \tag{16}$$

where $\Delta S_{BH} = S_{BH}(M-\omega, Q-q) - S_{BH}(M, Q)$ is the difference of Bekenstein-Hawking entropy, and $S_{BH} = \frac{r_+^n \operatorname{Vol}(S^n)}{4} = \frac{4\pi r_+^n}{n\omega_n}$ is the Bekenstein-Hawking entropy at the event horizon. Obviously, the derived emission spectrum actually deviates from pure thermality, and this result consists with an underlying unitary theory and perfectly generalizes those obtained by Parikh and Wilczek.

To end this discussion, we will discover the reason why the Parikh-Wilczek's semi- classical tunneling formalism is so successful that it satisfies the first law of the black hole thermodynamics. According to the definition of the surface gravity for the horizon, that is

$$k_+' = -\frac{1}{2}\lim_{r\to r_+'}\sqrt{\frac{-g^{11}}{g^{00}}}\frac{\partial}{\partial r}\ln(-g^{00}) = \frac{(r_+' - r_-')}{2r_+'^{2n-1}}, \tag{17}$$

so Equation (12) can be rewritten as

$$\begin{aligned}\operatorname{Im} S &= -\frac{1}{2}\int_{(M,Q)}^{(M-\omega,Q-q)} \frac{1}{T_+'}\left[d\left(M-\omega'\right) - \frac{n}{4(n-1)}\frac{\omega_n\left(Q-q'\right)}{r_+'^{(n-1)}}d(Q-q')\right] \\ &= -\frac{1}{2}\int_{S_{BH}(M,Q)}^{S_{BH}(M-\omega,Q-q)} dS' = -\frac{1}{2}\Delta S_{BH} = -\frac{2\pi}{n\omega_n}\left(r_f^n - r_i^n\right),\end{aligned} \tag{18}$$

where

$$T_+' = \frac{\kappa_+'}{2\pi} = \frac{(r_+' - r_-')}{4\pi r_+'^{2n-1}}, \tag{19}$$

is the Hawking temperature of the event horizon after charged particles emission. So from Equation (18), we can easily find the differential form of the first law of the black hole thermodynamics

$$dM' - A'_t dQ' = T'_+ dS', \tag{20}$$

where $M' = M - \omega'$ and $Q' = Q - q'$. Obviously, Parikh-Wilczek's semi-classical tunneling formalism is so successfully that it satisfies the first law of the black hole thermodynamics, and meanwhile proves that the factually radiant spectrum is not precisely thermal and the tunneling rate is related to Bekenstein-Hawking entropy, but is consistent with the underlying unitary theory and then the information conservation is possible. However, it is based on the reversible process, which is left to the following section.

4. DISCUSSION AND CONCLUSION

Although Parikh and Wilczek treated Hawking radiation as a tunneling process and gave a semi-classical but the first explicit calculation about the information conservation, the framework, which satisfies the first law of the black hole thermodynamics and consists with an underlying unitary theory ultimately attributed to the information conservation, is only suitable for the reversible process. Equation (20) is the differential form of the first law of the black hole thermodynamics, which is combined with the energy conservation law $dM - A_t dQ = dQ_h$ (where Q_h is the heat quantity) and the second law of the black hole thermodynamics $dS = dQ_h/T$ (where S is the entropy of the black hole). The equation of energy conservation is suitable for the reversible or the irreversible process, but the second law of the black hole thermodynamics $dS = dQ_h/T$ is only reliable for the reversible process. But in fact, the existence of the negative heat capacity, an evaporating black hole is a highly unstable system, and the thermal equilibrium between the black hole and the outside is unstable, there will be difference in temperature, so the process is irreversible, and for the moment the second law of the black hole thermodynamics should be $dS > (dQ_h/T)$. Thus the underlying unitary theory is not satisfied here, and therefore the information loss is possible during the evaporation, and the Parikh-Wilczek's tunneling framework can not prove the information conservation.

For another, the preceding study is still a semi-classical analysis, which means that the radiation should be treated as point particles. Such an approximation can only be valid in the low energy regime. If we are to properly address the information loss problem, then a better understanding of physics at the Planck scale is a necessary prerequisite, especially that of the last stages of Hawking evaporation.

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